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Developing cost-optimization production control model via simulation

Dimitri Golenko-Ginzburg[a,*], Ahazon Gonik[a], Ljubisa Papic[b]

[a]*Department of Industrial Engineering and Management, Ben-Gurion University of the Negev, Beer Sheva 84105, Israel*
[b]*DQM Research Center, P.O. Box 132, Cacak 32102, Serbia, Yugoslavia*

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Abstract

This research is a further development of our recent papers [2,3]. A production system produces a given target amount by a given due date and has several possible speeds which are subject to random disturbances. The system's output can be measured only at present inspection (control) points. The average manufacturing costs per time unit for each production speed and the average cost of performing a single inspection at the control point to observe the actual output at that point, are given. The least permissible probability of meeting the target on time, i.e. the system's chance constraint, is pre-given too. The problem is to determine both, control points and speeds to be introduced at those points, in order to minimize the system's expenses within the planning horizon. A stochastic optimization problem is formulated, followed by a heuristic solution via simulation. A numerical example is given. Extensive experimentation has been undertaken to illustrate the efficiency of the presented algorithm. The algorithm has been used in practice on a real man–machine plant. © 1999 IMACS/Elsevier Science B.V. All rights reserved.

Keywords: Control point; Chance constraint; Random production speeds; Cost objective; Heuristic solution via simulation

1. Introduction

In recent year extensive research has been undertaken in the area of production control models under random disturbances, especially on-line control models ([1–5,7–10], etc). These models usually refer to man–machine production systems with random parameters, e.g. construction, metallurgy and mining, research and development projects, for developing computer software and information systems, etc. For these not fully automatic systems, the actual output can be measured only at preset inspection (control) points. Such systems may normally use several possible speeds, which can be introduced by the decision-maker at control points. Given the target amount needed, the due date and the amount

* Corresponding author.

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produced up to a routine inspection point, the problem is to determine at that point the new production speed and the next inspection point. Two objectives have been usually embedded in the stochastic optimization model: (i) to minimize the number of inspection points, and (2) to maximize the probability of completing the production program on the due date.

However, the number of publications on *cost-optimization on-line control models under disturbances remains very scanty* (see, e.g. Ref. [3]), especially for control models under a chance constraint which have not been published elsewhere.

To fill the gap, we suggest a newly developed production control model which incorporates cost parameters. Two basic conceptions are embedded in the model:

A. The objective is to minimize the manufacturing expenses of completing the production program on the due date; and

B. A chance constraint, i.e. a confidence probability to meet the due date on time, has to be implemented in the model. In our opinion, such an additional restriction is important, since it guarantees the company's good name.

Thus, given:

– the target amount to be accomplished on time,
– the due date,
– several production speeds defined by their probability density functions,
– the average processing cost of realizing the manufacturing process per time unit for each production speed separately,
– the average cost of carrying out an inspection at the control point, and
– the chance constraint to meet the deadline on time,

the problem is to determine both, control points and production speeds to be introduced at each control point, to minimize the average manufacturing expenses within the planning horizon subject to the chance constraint.

This is a complicated stochastic optimization problem with a random number of decision variables, Since an optimal algorithm to solve the problem cannot be found, a heuristic one is suggested and developed. The algorithm is based on simulation and compares, one by one, sorted couples of production speeds in order to find an optimal couple that results in minimizing the average expenses. The algorithm has to be realized at any control point to choose both, the speed to be introduced and the next control point.

Note that all previous publications [2–5,10] are based on the *risk averse principle*, which is very efficient for non-cost objectives, but cannot be applied to the newly formulated cost-optimization model. It is therefore substituted for another one, namely the chance constraint principle, which is embedded in the heuristic algorithm and fits the cost objective.

The structure of the paper is as follows. In Section 2, the description of the production system is outlined. Section 3 considers Notations and presents the stochastic optimization problem. In Section 4, the chance constraint principle is described. In Section 5, the heuristic algorithm is outlined. Section 6 describes an illustrative numerical example, while in Section 7 extensive experimentation to verify the efficiency of the heuristic algorithm is presented. Practical applications of the algorithm on a real man–machine plant are outlined in Section 8. Conclusions and future research are presented in Section 9.

2. Description of the system

The system under consideration produces a single product or a production program that can be measured by a single value, just like the system described in [2], e.g. in percentages of the planned total volume. Such an approach is often used in R&D projects, in construction projects, in mining, etc. The system is subject to a chance constraint, i.e. the least permissible probability of meeting the due date on time is pre-given. The system utilizes non-consumable resources that remain unchanged throughout the planning horizon. There are several alternative processing speeds to realize the program, corresponding to the same given levels of resources and depending only on the degree of intensity of the production process. However, for different speeds, the average processing costs per time unit vary. The evaluation of advancing to the goal, i.e. observing the product's actual output, can be carried out only via timely inspections at pre-given control points. At every inspection (control) point, the decision-maker observes the amount produced and has to determine both, the proper speed and the next control point. Assume that it is prohibited to use unnecessarily high speeds (especially at the beginning of manufacturing the products), unless there is an *emergency situation*, i.e. a tendency to deviate from the target which may cause delay of the completion time. This is because lengthy work at higher speeds when utilizing restricted resources (e.g. manpower working in two or three shifts, etc.) can prematurely wear out the system. Assume, further, that the inspection and the speed-reset times are zero. The costs of all processing speeds per time unit, as well the cost of performing a single inspection at the control point, are pre-given.

3. The problem

Let us introduce the following terms:

V the production plan (target amount);
D the due date (panning horizon);

$V^f(t)$ the actual output observed at moment t, $0 < t \le D$; $V^f(0) = 0$;

$C^f(t)$ the actual accumulated processing and control costs calculated at moment t, $0 < t \le D$: $C^f(0) = 0$;
t_i the ith inspection moment (control point), $i = 0, 1, \ldots, N$;
N the number of control points (a random value);
V_j the jth speed, $1 \le j \le m$ (a random value with pregiven probability density function $f_j(v)$);
$\bar{v}_j$ the average speed v_j; it is assumed that speeds v_j are sorted in ascending order of the average values and are independent of t;
m the number of possible speeds;
s_i the index of the speed chosen by the decision-maker at the control point t_i;
c_j the average processing cost per time unit of speed v_j, $1 \le j \le m$ (pre-given); note that $j_1 < j_2$ results in $c_{j1} < c_{j2}$.
c_{ins} the average cost of performing a single inspection (pre-given);
Δ the minimal value of the closeness of the inspection moment to the due date (pre-given);

d the minimal given time span between two consecutive control points (in order to force convergence);
p the least permissible probability of meeting the due date on time (pre-given);
a_j lower bound of random speed v_j;
b_j upper bound of random speed v_j.
$W_p(t,j)$ the p-quantile of the moment production program V will be completed on condition that speed v_j is introduced at moment t and will be used throughout, and the actual observed output at that moment is $V^f(t)$ (time moment to be met with preset probability p); in other words, $W_p(t,j)$ is the p-quantile of the random value $\left[t + (V - V^f(t))/v_j\right]$.

Assume that values $V^f(t)$, as well as the parameters of the probability density functions $f_j(v)$, $1 \le j \le m$, are given in percentages of the planned target V. We will, henceforth, use three widely used distributions [2–6]:

1. a β-distribution with density function:

$$p_j(v) = \frac{12}{(b_j - a_j)^4}(v - a_j)(b_j - v)^2; \tag{1}$$

2. a uniform distribution in the same interval;
3. a normal distribution with the mean $\bar{v}_j = 0.5(a_j + b_j)$ and the variance $V_j = \left[(b_j - a_j)/6\right]^2$.

Let us consider the cost-optimization control problem. The problem is to determine both, control points $\{t_i\}$ and production speeds $\{s_i\}$ to minimize the manufacturing expenses:

$$J = \underset{\{t_i,s_i\}}{\text{Min}} \sum_{i=0}^{N-1} [c_{s_i}(t_{i+1} - t_i)] + N \cdot c_{\text{ins}} \tag{2}$$

such that:

$$\Pr\{V^f(D) \ge V\} \ge p, \tag{3}$$

$$t_0 = 0, \tag{4}$$

$$t_N = D, \tag{5}$$

$$t_{i+1} - t_i \ge d, \quad 0 \le i < N - 1, \tag{6}$$

$$D - t_i \ge \Delta, \quad 0 \le i \le N - 1, \tag{7}$$

$$s_i \le k = \min_{1 \le q \le m} [q : W_p(t_i, q) \le D]. \tag{8}$$

Objective (2) enables minimization of all manufacturing expenses, while objective (3) is the chance constraint. Eq. (4) implies that the first control point to undertake decision-making is zero—the starting moment to process the production program. Eq. (5) implies that the last inspection point is the due date D. Eq. (6) ensures the time span between two consecutive control points, while Eq. (7) provides the means of ensuring the closeness of the inspection moment to the due date. Eq. (8) means that the speed to be chosen at any routine control point t_i must not exceed *the minimal speed which guarantees* *meeting the deadline on time*, *subject to the chance constraint*. Thus, as outlined above, *unnecessary* surplus speeds are not introduced.

The problem defined in Eqs. (2)–(8) is a very complicated stochastic optimization problem which cannot be solved in the general case; it allows only a heuristic solution. The algorithm outlined below, in Section 5, determines at each control point t_i both, the next control point t_{i+1} and the speed v_{s_i} at which to proceed until that control point.

4. The chance constraint principle

The chance constraint principle is the basic approach for determining the next control point t_{i+1} on the basis of the routine control point t_i and the actual output $\mathrm{V}^{\mathrm{f}}(t_i)$ observed at that point. Note that such an approach has been successfully used [6] for controlling stochastic network projects.

Consider a routine control point t_i, together with the actual output observed at that point, $\mathrm{V}^{\mathrm{f}}(t_i)$. For each production speed v_j, $1 \leq j \leq m$, calculate via simulation a representative statistical sample $\{T_j^{(s)}\}$, where $T_j^{(s)}$ is the simulated value of the completion time of the production program obtained by using the speed v_j throughout. It is clear that the value of $T_j^{(s)}$ can be obtained from:

$$T_j^{(s)} = \frac{\mathrm{V} - \mathrm{V}^{\mathrm{f}}(t_i)}{v_j^{(s)}} + t_i, \tag{9}$$

where $v_j(s)$ is the simulated production speed v_j at the control point t_i.

After obtaining samples $\{T_j^{(s)}\}$, $1 \leq j \leq m$, calculate the corresponding p-quantiles $\mathrm{W}_p(t_i,j)$ and single out the subset of speeds for which:

$$\mathrm{W}_p(t_i,j) < \mathrm{D} \tag{10}$$

holds. Note that if, for a certain speed j, Eq. (10) holds, then all speeds with higher indexes also satisfy Eq. (10). Consider one of the speeds entering the subset, e.g. speed v_q. It is obvious (see Fig. 1) that, being introduced from point A (t_i, $\mathrm{V}^{\mathrm{f}}(t_i)$) throughout, speed v_q enables the deadline to be met on time, subject to the chance constraint. *Moreover*, *even if no processing at all takes place within the period of length* $\Delta t = \mathrm{D} - \mathrm{W}_p(t_i,q)$ (*see the straight line AF*) *and afterwards speed* v_q *is introduced at point F*, *this speed* v_q *still enables the deadline to be met on time*, *under the chance constraint* (Eq. (3)). This can be well-recognized by examining two parallel straight lines: line AE, which enables completion of the production program with a probability exceeding p (henceforth, call this line $\mathrm{AE}^{(q)}$) and the line BF which enables the deadline to be met on time with confidence probability equal to p (call this line $\mathrm{BF}^{(q)}$). Note that, if the production process proceeds with speed v_q from *any point* on line $\mathrm{BF}^{(q)}$, the target will be met on time subject to the chance constraint. This basic principle will be implemented in the heuristic algorithm.

5. The heuristic algorithm

Referring to Refs. [3,10], the heuristic control algorithm at each routine control point, t_i, enables minimization of the manufacturing expenses (Eq. (2)) during the *remaining time* $\mathrm{D} - t_i$. Thus, the objective function for optimizing decision-making at point t_i includes only *future expenses*, while past expenses, as well as past decision-makings, are considered to be irrelevant for the on-line control

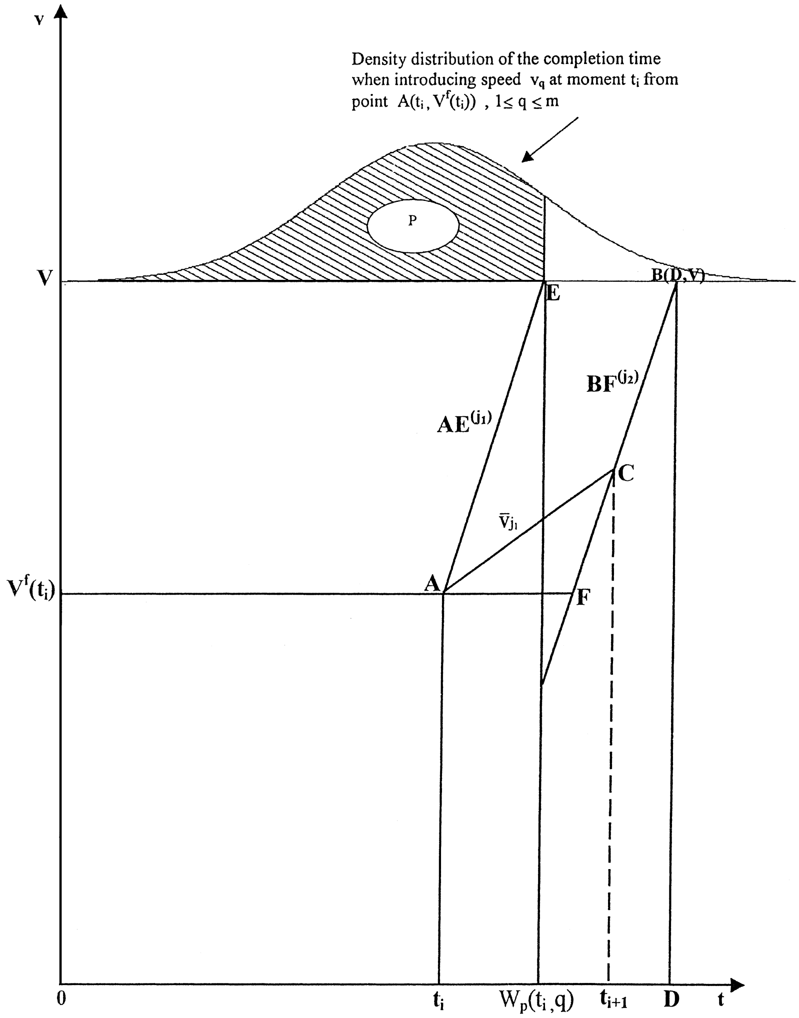

Fig. 1. The general idea of the chance constraint principle.

problem. At each control point t_i, decision-making centers around the assumption (see Refs. [3,10]) that there is no more than one additional control point before the due date.

It can be clearly seen that the backbone of the heuristic control algorithm is the *subalgorithm* I which, at each routine control point t_i determines both, the index s_i of the speed to be introduced and the next control point t_{i+1}. Following the assumption outlined above, two speeds have to be chosen at point t_i:

1. Speed v_{j_1}, $j_1 = s_i$, which *has to be actually introduced at point t_i up to the next control point* t_{i+1};
2. Speed v_{j_2}, $j_2 = s_{i+1}$, which *is forecast to be introduced at control point t_{i+1} within the period* $[t_{i+1}, D]$.

Note that, if speed j_2 is forecast to be the *last processing speed* before the due date D, control point t_{i+1} has to be on the straight line BF$^{(j_2)}$ (see Fig. 1), otherwise chance constraint (Eq. (3)) will not be met. We suggest singling out, at each routine control point t_i, all possible couples (j_1, j_2) satisfying restriction (Eq. (8)), and to choosing that one which delivers the minimum of forecasted manufacturing and control expenses, namely:

$$\min_{\{j_1,j_2\}}\{c_{j_1}(t_{i+1}-t_i)+c_{j_2}(\mathrm{D}-t_{i+1})+c_{\mathrm{ins}}\} \tag{11}$$

such that:

$$j_1 \le k = \min_{1\le q\le m}[q:\ \mathrm{W}_p(t_i,q)\le \mathrm{D}], \tag{12}$$

$$j_2 \ge k \text{ if } j_1 < k, \tag{13}$$

$$j_2 \le k \text{ if } j_1 = k. \tag{14}$$

Restriction (12) is embedded in the algorithm to satisfy restriction (8). Restriction (13) is true, since case $j_1 < k, j_2 < k$ contradicts chance constraint (3). Case $j_1 = k, j_2 > k$ is a pointless one since, for both cases (k, k) and $(k, j_2 > k)$, the chance constraint (3) will be met, but the second case is more costly.

As to value t_{i+1}, we suggest calculating the latter on the assumption that, being introduced at t_i, the actual processing speed is $\bar{v}_{j_1}$. Thus, t_{i+1} can be calculated as the abscissa of the intersection point C (see Fig. 1) of two straight lines:

$$\mathrm{AC}: v = \mathrm{V}^{\mathrm{f}}(t_i) + \bar{v}_{j_1}(t-t_i); \tag{15}$$

$$\mathrm{BV}^{(j_2)}: v = \frac{\mathrm{V}-\mathrm{V}^{\mathrm{f}}(t_i)}{\mathrm{W}_p(t_i,j_2)-t_i}t + \mathrm{V} - \mathrm{D}\frac{\mathrm{V}-\mathrm{V}^{\mathrm{f}}(t_i)}{\mathrm{W}_p(t_i,j_2)-t_i}. \tag{16}$$

Note that case $j_1 = j_2 = k$ is possible if using speed j_1 throughout, until the due date D, results in the cheapest realization. In such as case, value c_{ins} has to be excluded from Eq. (11).

The enlarged step-by-step procedure of the heuristic algorithm is as follows:

Step 1. Start with $i = 0$, $t_0 = 0$, $\mathrm{V}^{\mathrm{f}}(t_0) = 0$.

Step 2. For each speed v_j, $1 \le j \le m$, determine via simulation values $\mathrm{W_p}(t_i, j)$ (see Section 4).

Step 3. Determine $k = \min_{1\le q\le m}[q : \mathrm{W_p}(t_i,q) \le \mathrm{D}]$. If k cannot be determined, the problem defined in Eqs. (2)–(8) has no solution. Otherwise apply the next step.

Step 4. Consider couples as follows:

$$\begin{aligned}
&(1,k);(2,k);\ldots;(k-1,k);\\
&(1,k+1);(2,k+1);\ldots;(k-1,k+1);\\
&(1,k+2);(2,k+2);\ldots;(k-1,k+2);\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
&(1,m);(2,m);\ldots;(k-1,m);\\
&(k,k);(k,k-1);\ldots;(k,2);(k,1)
\end{aligned} \tag{17}$$

in accordance with restrictions (12–14).

Step 5. For each combination (j_1, j_2) of couples (17) calculate value t_{i+1} as abscissa of the intersection point of lines (15) and (16).

Step 6. For each combination of couples (17), check if $t_{i+1} - t_i \geq d$ holds. If not, calculate $t_{i+1} = t_i + d$.

Step 7. For each combination of couples (17), check if $t_{i+1} > \mathrm{D}$ or $\mathrm{D} - t_{i+1} < \Delta$. If one of these relations holds, set $t_{i+1} = \mathrm{D}$.

Step 8. For each combination of couples (17), besides $j_1 = k$ and $j_2 = k$, simulate the random forecasted output product:

$$\mathrm{V}(j_1, j_2) = v_{j_1} \cdot (t_{i+1} - t_i) + v_{j_2}(\mathrm{D} - t_{i+1}) + \mathrm{V}^{\mathrm{f}}(t_i) \tag{18}$$

many times in order to obtain representative statistics. In case $j_1 = j_2 = k$, the needed simulation has already been undertaken at steps 2 and 3. Assume (Ref. [2–6]) that any speed v_j is a *random* variable which, being simulated at the routine control point, remains unchanged until the next control point.

Step 9. Calculate for each pairwise combination (j_1, j_2) by means of statistical analysis, the approximate probability $\Pr\{\mathrm{V}(j_1, j_2) \geq \mathrm{V}\}$.

Step 10. Exclude from the list of combinations (17) all couples which satisfy $\Pr\{\mathrm{V}(j_1, j_2) \geq \mathrm{V}\} < p$.

Step 11. Add to the remaining list of combinations, the combination (k, k) and calculate for all those combinations the forecasted average costs $\mathrm{C}(j_1, j_2)$ as follows:

$$\mathrm{C}(j_1, j_2) = c_{j_1}(t_{i+1} - t_i) + c_{j_2}(\mathrm{D} - t_{i+1}) + c_{\mathrm{ins}} \text{ if } j_1 \neq j_2 \text{ and } t_{i+1} \neq D, \tag{19}$$

$$\mathrm{C}(j_1, j_2) = c_{j_1}(\mathrm{D} - t_i) \text{ if } j_1 \neq j_2 \text{ and } t_{i+1} = \mathrm{D}, \tag{20}$$

$$\mathrm{C}(k, k) = (\mathrm{D} - t_i) \text{ if } j_1 = j_2 = k. \tag{21}$$

Step 12. Consider the optimal couple (j_1, j_2) which delivers the minimum to $\mathrm{C}(j_1, j_2)$, If $j_1 \neq j_2$ and $t_{i+1} \neq \mathrm{D}$, go to the next step. If case (20) holds good, apply Step 16. For case (21) go to Step 15.

Step 13. Introduce speed j_1 up to the next control point t_{i+1}.

Step 14. Observe $\mathrm{V}^{\mathrm{f}}(t_{i+1})$ at moment t_{i+1}, and calculate value $\mathrm{C}^{\mathrm{f}}(t_{i+1}\ \mathrm{C}^{\mathrm{f}}(t_i) + (t_{i+1} - t_i)c_{j1} + c_{\mathrm{ins}}$; then set $i = i + 1$ and go to Step 2. Thus, at the next control point t_{i+1} subalgorithm I has to be realized anew.

Step 15. Introduce speed $j = k$ until $t_{i+1} = \mathrm{D}$. Go to Step 17.

Step 16. Introduce speed j_1 up to the due date D.

Step 17. Observe the output product at the due date D. Calculate $\mathrm{C^f(D)} = \mathrm{C^f}(t_i) + (\mathrm{D} - t_i)c_k + c_{\mathrm{ins}}$ in case (20).

End of the algorithm.

The algorithm is performed in real time, although it is based on statistical trials and simulation modeling. Each interaction of the algorithm can be done only after value $\mathrm{V^f}(t_i)$ is actually observed. However, the efficiency of the algorithm can be examined by simulating the actual speed v_{s_i} in each interval $[t_1, t_{i+1}]$. An illustrative numerical example and extensive experimentation will be outlined below.

6. Numerical example

Let us consider the following example: a production system with five possible speeds uniformly distributed:

$$v_1 = U(1.8, 2.3);\ c_1 = 10;$$

$$v_2 = U(2, 2.5);\ c_2 = 20;$$

$$v_3 = U(2.5, 3.1);\ c_3 = 40;$$

$$v_4 = U(3, 3.4);\ c_4 = 50;$$

$$v_5 = U(3.5, 4);\ c_5 = 60.$$

Other parameter are:

$$\mathrm{V} = 77,\ \mathrm{D} = 30,\ p = 0.75,\ c_{\mathrm{ins}} = 40,\ d = 3,\ \Delta = 3. \quad \square$$

The solution according to the heuristic algorithm outlined above (for one simulation run) is as follows:

Stage I. $i = 0$, $t_0 = 0$, $\mathrm{V^f}(0) = 0$.
Values $\mathrm{W}_p(0, j)$ obtained via simulation (Step 2) are:

$$\mathrm{W}_p(0,1) = 40,\ \mathrm{W}_p(0,2) = 36.3,\ \mathrm{W}_p(0.3) = 29.1,\ \mathrm{W}_p(0,4) = 24.9,\ \mathrm{W}_p(0,5) = 21.2.$$

Thus, $k = 3$ (see Step 3 of the heuristic algorithm), and the possible couples (j_1, j_2) to be examined (see Step 4) are as follows:

$$(1,3),\ (1,4),\ (1,5),\ (2,3),\ (2,4),\ (2,5),\ (3,3),\ (3,2),\ (3,1).$$

Values t_{1+d} and Pr $\{\mathrm{V}(j_1, j_2) \geq \mathrm{V}\}$ for each couple (j_1, j_2) are calculated by using (Eqs. (15) and (16)) (see Step 5) and (Eq. (18)) (see steps 8–9), respectively:

Couple (j_1,j_2)	Value t_1	Value $\Pr\{V(j_1,j_2) \geq V\}$
(1,3)	3.871	0.759
(1,4)	15.077	0.721
(1,5)	20.224	0.582
(2,3)	5.840	0.772
(2,4)	18.672	0.643
(2,5)	23.143	0.563
(3,3)	30	>0.75 (not calculated, since $W_p(0,3) < 30$
(3,2)	19.664	0.622
(3,1)	21.994	0.556

Thus, only couples (1,3), (2,3) and (3,3) are left after Step 10 for further examination.

The corresponding average processing costs $C(j_1, j_2)$ obtained in Step 11 by (Eqs. (19)–(21)) are as follows:

$$C(1,3) = 1123.87$$

$$C(2,3) = 1123.20$$

$$C(3,3) = 1200.$$

Thus, pair (2,3) has to be chosen at Step 12.

The simulated speed of v_2 (see Step 13) from $t_0 = 0$ up to $t_1 = 5.84$ is 2.3311. The simulated output at the next control point $V^f(t_1) = 13.61$, while the calculated processing and control costs are $5.84 \cdot 20 + 40 = 156.80$.

Stage II. $i = 1$, $t_1 = 5.84$, $V^f(t_1) = 13.61$.

Values $W_p(t_1,j)$ obtained via simulation (see Step 12) are $W_p(t_1,1) = 38.61$, $W_p(t_1,2) = 35.76$, $W_p(t_1,3) = 29.85$, $W_p(t_1,4) = 26.30$, $W_p(t_1,5) = 23.30$. Thus $k = 3$, and the couples (j_1, j_2) delineated above have to be examined anew. The corresponding values t_2 and $\Pr\{V(j_1, j_2) \geq V\}$ have been calculated at point $t_1 = 5.84$. Note that, since the use of relations (Eqs. (15) and (16)) for couples (1,3) and (2,3) results in values $t_2 = 6.52$ and $t_2 = 6.87$, respectively, $t_2 - t_1 < d$ and values t_2 have been updated according to Step 6. Thus, for both couples (1,3) and (2,3), values $t_2 = t_1 + 3 = 8.84$. Values t_{i+1} and $\Pr\{V(j_1, j_2) \geq V\}$ for all couples (j_1, j_2) under examination are as follows:

Couple (j_1, j_2)	Value t_2	Value $\Pr\{V(j_1, j_2) \geq V\}$
(1,3)	8.84	0.749
(1,4)	16.77	0.722
(1,5)	21.23	0.635
(2,3)	8.84	0.746
(2,4)	19.35	0.660
(2.5)	23.46	0.618
(3.3)	30	>0.75 (not calculated), since $W_p(t_1,3) < 0$
(3,2)	23.75	0.550
(3,1)	25.08	0.545

Thus, only couple (3,3), which results in $\Pr\{V(3,3) \geq V\} > 0.75$, remains after Step 12. The simulated speed v_3 at Step 13 and up to the date D = 30 is $v_3 = 2.7325$.

Stage III. $t_2 = D = 30$. The actual simulated output at moment $t_2 = 30$ is $V^f(30) = 13.61 + 2.7325 \times (30 - 5.84) = 79.63 > 77$.

The actual accumulated total expenses are $156.80 + 40(30 - 5.84) + 40 = 1163.2$. Thus, the system has met its target on time with two inspection points.

7. Experimentation

In order to evaluate the performance of the algorithm, various examples were run. The experimental design is given in Table 1. Other parameters are similar to those outlined above, in Section 6. Three variables are varied, namely, p, c_{ins} and the distribution of v_j. The number of possible speeds m is fixed and equals five. The target amount, the due date and the processing costs, c_j, $1 \leq j \leq 5$, are fixed and remain unchanged. A total of 144 combinations were considered. For each combination, 1000 runs were carried out. Several measures were considered as follows:

$\bar{C}$ the optimal average value of total expenses within one simulation run.
$\bar{P}$ the average actual probability of meeting the due date on time;
$\bar{N}$ the average number of inspection points (without t_0).
$\bar{J}$ the average index of the speed to be introduced within one simulation run.

Value $\bar{J}$ for one simulation run is calculated as follows:

$$\bar{J} = \frac{1}{D} \sum_{i=0}^{N-1} [(t_{i+1} - t_i) s_i].$$

We have implemented the algorithm on an IBM PC in the Tuzbo Pascal programming language.

The summary of simulation experiments is shown in Table 2 (uniform distribution), Table 3 (normal distribution) and Table 4 (β-distribution).

The following conclusions can be drawn from the summary:

1. The average number of inspection points for all types of distributions is quite low—between one and two, including the due-date count—while the number of intermediate inspection points is actually <1.

Table 1
The experimental design

Variable	Values given in the experiments	Number of values
The average cost of a single inspection c_{ins}	10;20;30;40;60;100	6
The least permissible probability p in the chance constraint	0.60;0.65;0.70;0.75 0.80;0.85;0.90;0.95	8
Distribution of v_j	uniform, normal, beta	3

Table 2
Simulation experiments for the uniform distribution

Initial values		Outcome values			
C_{ins}	P	$\bar{C}$	$\bar{P}$	$\bar{N}$	$\bar{J}$
10	0.60	1133	0.840	1.672	1.921
	0.65	1161	0.839	1.676	1.981
	0.70	1182	0.862	1.677	2.016
	0.75	1195	0.882	1.743	2.049
	0.80	1180	0.857	1.819	1.943
	0.85	1209	0.892	1.073	1.997
	0.90	1237	0.967	1.674	2.067
	0.95	1250	0.995	2.000	2.099
20	0.60	1156	0.863	1.666	1.947
	0.65	1183	0.858	1.670	1.996
	0.70	1202	0.851	1.674	2.029
	0.75	1210	0.853	1.747	2.039
	0.80	1196	0.855	1.871	1.940
	0.85	1220	0.904	1.077	1.998
	0.90	1253	0.953	1.663	2.066
	0.95	1270	0.996	2.000	2.099
30	0.60	1164	0.861	1.692	1.916
	0.65	1192	0.851	1.681	1.971
	0.70	1219	0.841	1.655	2.028
	0.75	1224	0.870	1.774	2.027
	0.80	1217	0.866	1.845	1.945
	0.85	1230	0.901	1.071	1.998
	0.90	1270	0.957	1.673	2.067
	0.95	1290	0.992	2.000	2.099
40	0.60	1183	0.861	1.666	1.923
	0.65	1214	0.853	1.643	1.987
	0.70	1236	0.861	1.656	2.028
	0.75	1246	0.854	1.774	2.044
	0.80	1235	0.842	1.869	1.942
	0.85	1241	0.876	1.079	1.998
	0.90	1286	0.946	1.660	2.066
	0.95	1310	0.988	2.000	2.099
60	0.60	1225	0.841	1.682	1.942
	0.65	1241	0.852	1.688	1.968
	0.70	1278	0.871	1.662	2.059
	0.75	1277	0.870	1.751	2.031
	0.80	1270	0.874	1.834	1.939
	0.85	1263	0.884	1.084	1.998
	0.90	1319	0.957	1.656	2.065
	0.95	1350	0.997	2.000	2.099
100	0.60	1285	0.868	1.676	1.928
	0.65	1311	0.853	1.694	1.976

Table 2 (*Continued*)

Initial values		Outcome values			
C_{ins}	P	$\bar{C}$	$\bar{P}$	$\bar{N}$	$\bar{J}$
	0.70	1341	0.866	1.673	2.046
	0.75	1351	0.855	1.721	2.046
	0.80	1301	0.887	1.006	2.001
	0.85	1300	0.890	1.000	2.000
	0.90	1386	0.937	1.659	2.065
	0.95	1430	0.993	2.000	2.099

Table 3
Simulation experiments for the normal distribution

Initial values		Outcome values			
C_{ins}	P	$\bar{C}$	$\bar{P}$	$\bar{N}$	$\bar{J}$
10	0.60	1142	0.917	1.487	1.910
	0.65	1157	0.919	1.447	1.939
	0.70	1164	0.925	1.424	1.946
	0.75	1171	0.940	1.415	1.962
	0.80	1174	0.966	1.422	1.965
	0.85	1173	0.959	1.462	1.958
	0.90	1183	0.971	1.530	1.987
	0.95	1187	0.987	1.687	1.992
20	0.60	1153	0.897	1.488	1.893
	0.65	1171	0.907	1.472	1.942
	0.70	1178	0.943	1.382	1.939
	0.75	1182	0.939	1.424	1.953
	0.80	1185	0.971	1.409	1.953
	0.85	1193	0.963	1.446	1.973
	0.90	1202	0.968	1.508	1.995
	0.95	1201	0.969	1.684	1.983
30	0.60	1174	0.892	1.483	1.918
	0.65	1181	0.918	1.458	1.926
	0.70	1192	0.924	1.406	1.943
	0.75	1193	0.957	1.429	1.941
	0.80	1195	0.947	1.451	1.941
	0.85	1207	0.959	1.460	1.974
	0.90	1215	0.975	1.542	1.992
	0.95	1221	0.980	1.698	1.993
40	0.60	1193	0.908	1.487	1.926
	0.65	1201	0.920	1.436	1.937
	0.70	1202	0.927	1.448	1.935
	0.75	1214	0.946	1.410	1.963
	0.80	1218	0.959	1.448	1.971
	0.85	1224	0.953	1.476	1.982
	0.90	1226	0.953	1.530	1.979
	0.95	1232	0.975	1.678	1.974

Table 3 (*Continued*)

Initial values		Outcome values			
C_{ins}	P	$\bar{C}$	$\bar{P}$	$\bar{N}$	$\bar{J}$
60	0.60	1216	0.906	1.494	1.906
	0.65	1224	0.919	1.474	1.923
	0.70	1238	0.929	1.386	1.953
	0.75	1243	0.953	1.400	1.964
	0.80	1241	0.954	1.439	1.952
	0.85	1245	0.961	1.448	1.954
	0.90	1258	0.974	1.519	1.980
	0.95	1271	0.972	1.690	1.991
100	0.60	1278	0.908	1.516	1.917
	0.65	1287	0.937	1.456	1.936
	0.70	1292	0.938	1.423	1.947
	0.75	1293	0.947	1.460	1.939
	0.80	1300	0.955	1.413	1.958
	0.85	1308	0.966	1.449	1.970
	0.90	1314	0.971	1.536	1.963
	0.95	1326	0.985	1.442	1.997

Table 4
Simulation experiments for the beta distribution

Initial values		Outcome values			
C_{ins}	P	$\bar{C}$	$\bar{P}$	$\bar{N}$	$\bar{J}$
10	0.60	1181	0.844	1.657	2.028
	0.65	1200	0.864	1.638	2.065
	0.70	1208	0.862	1.631	2.073
	0.75	1174	0.861	1.785	1.936
	0.80	1181	0.893	1.823	1.944
	0.85	1199	0.930	1.497	1.980
	0.90	1210	0.943	1.000	2.000
	0.95	1246	0.989	1.887	2.088
20	0.60	1203	0.848	1.666	2.050
	0.65	1207	0.872	1.661	2.037
	0.70	1213	0.878	1.659	2.034
	0.75	1196	0.878	1.788	1.952
	0.80	1201	0.889	1.820	1.951
	0.85	1214	0.921	1.549	1.981
	0.90	1220	0.929	1.000	2.000
	0.95	1264	0.988	1.881	2.088
30	0.60	1211	0.844	1.691	2.026
	0.65	1227	0.856	1.649	2.044
	0.70	1238	0.881	1.635	2.063
	0.75	1210	0.860	1.853	1.931
	0.80	1217	0.878	1.801	1.947

Table 4 (*Continued*)

Initial values		Outcome values			
C_{ins}	P	$\bar{C}$	$\bar{P}$	$\bar{N}$	$\bar{J}$
	0.85	1228	0.919	1.491	1.980
	0.90	1230	0.932	1.000	2.000
	0.95	1283	0.991	1.891	2.089
40	0.60	1224	0.850	1.674	2.016
	0.65	1248	0.875	1.659	2.063
	0.70	1254	0.853	1.654	2.060
	0.75	1227	0.857	1.821	1.934
	0.80	1236	0.868	1.826	1.946
	0.85	1243	0.928	1.525	1.978
	0.90	1240	0.929	1.000	2.000
	0.95	1302	0.993	1.899	2.089
60	0.60	1260	0.844	1.689	2.017
	0.65	1285	0.851	1.673	2.075
	0.70	1289	0.850	1.645	2.068
	0.75	1267	0.860	1.794	1.946
	0.80	1272	0.887	1.839	1.946
	0.85	1274	0.919	1.526	1.979
	0.90	1260	0.931	1.000	2.000
	0.95	1340	0.988	1.898	2.089
100	0.60	1329	0.842	1.669	2.031
	0.65	1346	0.863	1.628	2.061
	0.70	1350	0.875	1.660	2.049
	0.75	1340	0.878	1.784	1.954
	0.80	1300	0.934	1.000	2.000
	0.85	1300	0.943	1.000	2.000
	0.90	1300	0.926	1.000	2.000
	0.95	1417	0.991	1.901	2.090

2. The average actual probability $\bar{P}$ of meeting the due date on time for all types of distributions exceeds its corresponding pregiven chance constraint value p. Thus, the control algorithm minimizes value $\bar{C}$ with respect to Eq. (3).
3. Using a normal distribution yields the highest probability values $\bar{P}$, while using a β-distribution leads to the lowest values.

8. Practical applications

In 1997–1998, extensive experimentations were undertaken in several industrial plants in Serbia to test the fitness of the developed cost-optimization production control model. The best results were achieved in a Serbian company ‘Hidrogradnja’ in Cacak (Belgrade district), which specializes in the design and construction of unique installations, e.g. the railway bridge over the river Morava, the unique railroad tunnel 'Bukovi' through a mountain, various power stations, etc. 'Hidrogradnja' includes a man–machine production plant which works under random disturbances. In order to maintain the company's good name and to compete with other companies, the projects' deadlines have to be met on time, subject to chance constraint. The planning horizons for building similar installations are usually more than a year, while the chance constraint to meet the deadline on time is about $p = 0.8$. Since 'Hidrogradnja' can work with one, two and three shifts, the number of possible speeds has been taken to be three for all experiments.

Evaluating the project's accomplishments continuously in building construction undertaken by 'Hidrogradnja' is difficult and costly, so periodic inspections are preferred. The actual output observed at a current inspection point is measured in percentages of the total project (target amount). When realizing a certain project in 'Hidrogradnja' for all teams working simultaneously on that project, the shift has to be of equal length. This enables use of the algorithm outlined above, which has been applied to designing and constructing two installations in 1997–1998, namely:

(a) a power station; and
(b) a new railway bridge over river West Morava.

In the course of the control model's installation, the measurement was faced with developing a simulation model comprising the probability parameters for all production speeds. The minimal time span between two consecutive inspection points was set equal to 20 days.

By using the newly-developed model, the management successfully met the corresponding due dates on time for both the installations, unlike many similar projects which have been realized before through traditional, off-line control models. The operational and control expenses decreased by 7–8%. Thus, the suggested control model can be applied to a broad spectrum of construction systems under random disturbances.

9. Conclusions and future research

1. The on-line cost-optimization control algorithm, developed here, can be used for various man–machine production systems which can be operated at several different speeds subject to random disturbances, for which a partial accomplishment can be measured as a part of the whole planned program. Such systems include construction projects, various R&D projects, etc. The system's output can be measured only at pregiven inspection points, since it is impossible or too costly to measure it continuously.
2. The control algorithm is carried out via simulation and can be easily programmed on PC.
3. On-line control algorithm with a cost objective subject to a chance constraint is a further development of broad variety of control algorithms with non-cost parameters. Such an algorithm is, in essence, a linkage between cost- and non-cost objectives.
4. The cost-optimization control algorithm performs well. The algorithm minimized the average value of the total expense by using a very low number of inspection points with respect to the chance constraint.
5. The on-line control model developed here has been applied successfully for designing and creating new machines and installations on a real industrial plant.

6. Future research may be undertaken to develop multilevel production control models under random disturbances with chance constraints to be embedded in models at each hierarchical level. However, such a problem can be solved by taking into account multilevel interaction and coordination methods [8].

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