

# Mineral investment valuation and the cost of capital

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**The author discusses some of the mineral investment models used. He outlines the theoretical foundations of the cost of capital concept and derives models for determining the cost of capital from specific sources of financing. The use of the weighted average cost of capital appears to have advantages and is illustrated with an example.**

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<sup>1</sup> See H.D. Hoskold, *The Engineering Valuing Assistant*, Longmans, Green & Co, 1877, 1905.

<sup>2</sup> See D.B. Morkill, 'Formulas for mine valuation,' *Mineral & Scientific Press*, Vol 117, (1918), p 276.

<sup>3</sup> To arrive at the Hoskold formula let

- $r$  = risky rate of return on invested capital;
- $f$  = riskless (or safe) rate of return on reinvested capital redemption;
- $E$  = annual earnings, assumed to be equal over the horizon;
- $n$  = life of project;
- $V_p$  = present value of future income, or today's investment;
- $rV_p$  = required risky return of investment in dollars;
- $(E-rV_p)$  = annual redemption assumed to be invested periodically, as a sinking fund, at an interest rate of  $f$ , to yield the amount of capital originally invested;

*continued p.285*

With the changing nature of the world economy, and the scarcity of mineral resources, mineral valuation becomes more and more important. Investment in mineral industries, in general, and in the mining industry in particular, differs from the ordinary industry in that most industries and enterprises have an indeterminate life, assumed to be perpetual, and therefore are not called on to replace the original investment. In mining ventures, on the other hand, once the ore bodies are mined out, there is nothing left. The original capital investment in a particular property must, therefore, be returned to the investor by the time the ore is exhausted at which time the mine has no economic value.

A mining venture is also characterised by the significant cost and time associated with the exploration phase; many samples are usually taken; investigation is made regarding the geological features of the deposit; and the grade of ore is studied. In addition, most mining ventures require substantial capital expenditures aggregating several hundred million dollars for a number of years before the project begins producing any revenue. Any theoretical model employed in evaluating a mining venture, as well as similar mineral ventures, must take all these factors into consideration.

## Mineral investments valuation models

**The Hoskold and Morkill models.** Historically, mining engineers were satisfied in using the Hoskold formula<sup>1</sup> or the Morkill approach.<sup>2</sup> The underlying assumption of the Hoskold method is that the firm wants to obtain a return on its investment while at the same time recover the cost of its investment. In other words, the project must yield a speculative return and also pay annual cash payments to a sinking fund, invested at a safe rate of return, to yield the initial investment at the exhaustion of the project.<sup>3</sup>

The Hoskold method has been criticised for three related deficiencies.<sup>4</sup> The first is that employment of this formula results in an undervaluation of mineral properties as compared to the ordinary discounting procedures for valuing future receipts. The undervaluation stems from the employment of two significantly different rates of return: a safe rate, and a risky rate. The larger the difference between these two rates the more undervalued a mineral

$F_n$  = compound value of annuity of \$1 per year for  $n$  years at the riskless rate of return  $f$ ;  
 $R^n = (1 + f)^n$ .

Then

$$V_p = (E - rV_p) F_n \quad (1)$$

$$= EF_n - rV_p F_n \quad (2)$$

$$V_p (1 + rF_n) = EF_n \quad (3)$$

and

$$V_p = \frac{EF_n}{(1 + rF_n)} \quad (4)$$

Dividing both the numerator and denominator by  $F_n$  yields

$$V_p = \frac{E}{\frac{1}{F_n} + r} \quad (5)$$

It is also known that (verification can be found in any finance text)

$$F_n = \frac{(1 + f)^n - 1}{f} \quad (6)$$

$$= \frac{R^n - 1}{f} \quad (7)$$

Equation 5 can, therefore, be rewritten as

$$V_p = \frac{E}{\frac{f}{R^n - 1} + r} \quad (8)$$

Equation 8 is the Hoskold formula.

<sup>4</sup> See for example, Douglas H. Eldridge, 'Is use of the Hoskold formula justified?' *Engineering and Mining Journal*, August 1969, pp 72-74.

<sup>5</sup> The derivation of the Morkill formula can be found in D.B. Morkill, 'Formulas for mine valuation,' *Mineral & Scientific Press*, Vol 117, p 276.

<sup>6</sup> See for example, George F. Weaton, 'Valuation of mineral deposits,' *Mining Engineering*, May 1973, pp 29-36.

<sup>7</sup> See for example, Emory J. Douglas, 'How to make the most of a mining investment,' *Mining Engineering*, October 1971, pp 64-67.

<sup>8</sup> See for example, Raymond F. Mikesell, 'Financial considerations in negotiating mine development agreements,' *Mining Magazine*, April 1974, pp 257-271.

deposit may be as compared to the value derived by ordinary capitalisation procedures. The second argument against the formula is that mineral enterprises do not, in practice, set up a sinking fund in order to recoup the original investment. It is argued that the goal of a management of a mineral enterprise, as well as any other enterprise, is to maximise the yield from capital funds. In doing so, they do not invest in a sinking fund at a safe rate, but rather keep their funds employed where expected gains are most promising. The elimination of the sinking fund also removes the use of two rates of return, and consequently, does not undervalue the mineral property. Finally, under the Hoskold method the risky return is calculated on the initial investment and is not reduced as the investment is recovered.

The Morkill formula for a mineral property valuation is as follows

$$V_p = \frac{E[(1 + r)^n - 1]}{(1 + r)^n r} \quad (9)$$

where the terms are as defined earlier (Ref 3).<sup>5</sup> Morkill agrees with Hoskold that the total return from a mineral investment should be separated into two parts: a speculative (risky) return, and a safe return to be invested in a sinking fund so as to yield the initial investment at the exhaustion of the project. In other words, Morkill supports the employment of two different rates of return. Morkill, however, disagrees with Hoskold's method of apportionment, and maintains that the risky rates of return should be expected only from the amount of capital remaining unrecovered, and that, therefore, the two parts which constitute total returns should not be constant over time. If we assume that the annual total returns from the project remain constant over the life of the project, then, the returns associated with the risky component decrease over time, and the amount of funds allotted to the sinking fund increase.

This method is being attacked both for having two different rates of return, and for advocating a sinking fund arrangement. Although most mining engineers and other professionals involved with the mineral industry are aware of the criticisms of the Hoskold and Morkill methods their use is still widespread.<sup>6</sup> One explanation may be the reluctance to change a system once adopted. Nevertheless, in recent years new and more sophisticated approaches have emerged and the mineral industry seemed to welcome them. Among these approaches we find the net-present-value approach (NPV)<sup>7</sup> and the discount cash flow approach (DCF), also known as the internal-rate-of-return (or IRR).<sup>8</sup>

### The NPV and DCF methods

With the net present value method, all cash flows are discounted to present value using the cost of capital (which is also defined as the minimum required rate of return on an investment). The net present value of an investment proposal is

$$NPV = \sum_{t=0}^n \frac{A_t}{(1 + r)^t} \quad (10)$$

where

$A_t$  = cash flow for period  $t$ , whether it be a net cash outflow or inflow;

$r$  = cost of capital for the firm;

$n$  = last period in which a cash flow is expected.

It should be noted that under conditions of uncertainty, or risk, the cost of capital should account for risk by including an appropriate risk premium. If the sum of these discounted cash flows is greater than 0, the proposal is accepted; if it is less than 0, the proposal is rejected. In other words, if  $NPV > 0$ , the proposal is accepted; if  $NPV < 0$  it is rejected; and if  $NPV = 0$ , the investor is indifferent as to whether or not to undertake the project.

The internal-rate-of-return (the DCF rate) for an investment proposal is the discount rate that equates the present value of expected cash outflows with the present value of expected cash inflows. The IRR for an investment proposal is determined by setting

$$\sum_{t=0}^n \frac{A_t}{(1+\rho)^t} = 0 \quad (11)$$

and solving for  $\rho$ , which is the internal-rate-of-return. The other variables are the same as defined earlier. Equation 11 suggests that  $\rho$  is found at the point where the present value of all cash outflows (cost) discounted by  $\rho$  are equal to the present value of all cash inflows discounted by  $\rho$ , or that the NPV is equal to 0. To obtain  $\rho$  we have to solve the polynomial described in Equation 11.<sup>9</sup> This can easily be done by means of a computer or by a manual trial and error method. The acceptance-rejection decision rule of the IRR approach is based on the comparison between the IRR obtained for the proposed project and the firm's cost of capital,  $r$ . If  $\rho > r$ , the project should be accepted, if  $\rho < r$ , the project is rejected; and if  $\rho = r$ , the decision maker should be indifferent as to whether or not to accept or reject the project.

It should be noted that there may be instances in which a conflict between these two methods occurs. This conflict is due to the different assumptions with respect to the marginal reinvestment rates on funds released from the proposal. The IRR method implies that funds are reinvested at the internal-rate-of-return over the remaining life of the project. The NPV method, on the other hand, implies reinvestment at a rate equivalent to the minimum required rate of return (cost of capital) used as the discount rate.<sup>10</sup> From a theoretical standpoint the net-present value approach is considered superior to the internal-rate-of-return.<sup>11</sup>

All four investment valuation criteria rely critically on the information concerning the cost of capital or the minimum required rate of return. With the Hoskold and the Morkill methods the risky rate of return is the minimum required rate of return, namely, the cost of capital associated with investing funds in the mineral property. It should also be noted, at the outset, that the discussion of the NPV and IRR methods indirectly supports the criticism associated with the employment of a risk-free rate to that component of the return being invested in a sinking fund. This is so because both the NPV and IRR methods implicitly assume risky reinvestment rates, which are applied to the funds released by the project: the cost of capital,  $r$ , in the NPV approach, and the internal-rate-of-return,  $\rho$ , in the IRR approach.

<sup>9</sup>When solving for  $\rho$  it is important to recognise that the polynomial may yield more than one real solution for  $\rho$ . In other words, there may be more than one internal-rate-of-return that equates the present value of cash inflows with the present value of cash outflows. This can occur when there are net cash outflows in more than one period and the outflows are separated by one or more periods of new cash inflows. It should be noted that the existence of multiple internal-rates-of-return is not the rule but rather the exception. For the typical capital budgeting project a unique internal-rate-of-return usually exists.

<sup>10</sup>See Ezra Solomon, *The Theory of Financial Management*, Columbia Press, New York, 1963, pp 134-135.

<sup>11</sup>J. Hirshleifer, 'On the theory of optimal investment decision,' *Journal of Political Economy*, Vol 66, August 1958, pp 95-103; and James H. Lorie and Leonard J. Savage, 'Three problems in rationing capital,' *Journal of Business*, Vol 23, October 1955, pp 299-39.

Under conditions of uncertainty the cost of capital will always be greater than the riskless rate. Consequently, neither approach would consider investing part of the funds released by the project at a safe rate, regardless of whether or not a sinking fund is advocated.

It is surprising to note that although the NPV and the IRR are often recommended for use in the mineral resources sector, their practical application fails to recognise the important and crucial role of the cost of capital. As mentioned earlier, the cost of capital determines acceptance or rejection of an investment proposal. Emory J. Douglas, for example, while suggesting that the net-present value approach should be used states that

‘Projects should be evaluated at the lowest acceptable rate of return or in other words, the highest unacceptable rate of return. This rate needs to be a real fixed percent of interest which divides acceptable from unacceptable investments. Its level is one of the more important decisions for top management.’<sup>12</sup>

This quotation referring to the cost of capital as ‘a real fixed percent of interest,’ demonstrates much of the confusion associated with the cost of capital concept. In some cases the minimum risky rate of return, ie, the cost of capital, is assigned an arbitrary value such as 8-12%.<sup>13</sup>

In another example, Raymond Mikesell advocates the employment of the internal-rate-of-return approach.<sup>14</sup> However, he does not consider the crucial role the cost of capital should play in determining the minimum acceptable internal-rate-of-return for a firm, but rather simply assumes that this cut-off level falls somewhere between 12-20%.<sup>15</sup>

Examination of the ability of the project to generate sufficient funds so as to meet payments of fixed commitments only, ie, interest payments as well as payment of principal, fails to take into account the cost of equity financing and the additional financial risk incurred when a firm is using more and more debt financing.<sup>16</sup> It is, therefore, of paramount importance that in evaluating a mineral venture the overall cost of capital is used.

<sup>12</sup> Emory J. Douglas, ‘How to make the most of a mining investment,’ *Mining Engineering*, October 1971, p 64.

<sup>13</sup> George F. Weaton, ‘Valuation of mineral deposits,’ *Mining Engineering*, May 1973, p 37.

<sup>14</sup> See Raymond F. Mikesell, *Foreign Investment in Copper Mining*, Resources for the Future, Inc, The Johns Hopkins University Press, Baltimore, 1975, p 5; and Raymond F. Mikesell, ‘Financial consideration in negotiating mine development agreement,’ *Mining Magazine*, April 1974, p 259.

<sup>15</sup> Raymond F. Mikesell. *Foreign Investment in Copper Mining*, op cit Reference 14, pp 17-18 and p 27.

<sup>16</sup> Financial risk is added to business risk when a firm instead of using all equity financing obtains a part of its capital through the issuance of debt. This added risk is due to the additional burden imposed on management to meet its fixed obligations, ie, interest payments, etc, regardless of its profit position. Also, in a case of liquidation, creditors are considered preferred claimants against the assets and the income of the firm. Once they are satisfied, the shareholders may present a claim against the balance. The existence of creditors, therefore, increases the risk on the part of the shareholders.

### **Cost of capital: some theoretical foundations**

The investment decision of a firm is related directly to the financing decision because the acceptance of investment proposals depends upon how those proposals will be financed. Furthermore, the hurdle rate used to allocate funds among competing investment proposals, ie, the cost of capital, is the minimum rate of return that a proposal must generate so as to leave unchanged the market price of the stock. In general, investments should be undertaken only if they can promise a rate of return higher than the cost of capital.

The cost of capital for a firm is a combination of two components: the explicit cost, and the implicit cost. The explicit cost for the utilisation of any source of funds is the discount rate that equates the present value of inflows and outflows associated with a financing instrument. In other words, the explicit cost of a source of financing is the discount rate that equates the present value of the funds received by the firm from that source, net of underwriting and other costs, with the present value of expected outflows. These outflows may be in terms of dividends, interest payments on loans and bonds, or a

repayment of principal. The explicit cost of capital may be determined by solving the following equation for  $r$ :<sup>17</sup>

$$L_0 = \frac{c_1}{(1+r)} + \frac{c_2}{(1+r)^2} + \dots + \frac{c_n}{(1+r)^n} \quad (12)$$

where

- $L_0$  = net amount of funds received by the firm at time 0;
- $c_t$  = cash outflow in period  $t$ ; and
- $n$  = horizon over which the funds are provided.

It should be noted that since the cost of capital is used in deciding whether to invest in new projects, the concern must be with raising new or incremental capital to satisfy the project requirements, rather than with capital raised in the past. In other words, the concern must be with determining the marginal explicit cost of a specific method of financing, ie, the cost associated with raising an additional dollar by the financing method in question. Past costs of financing have no bearing on this decision.<sup>18</sup>

The implicit cost of capital comes into play when a firm is faced with capital rationing, ie, its available funds for investment are limited. In such a case, the firm is not able to undertake all investments available to it. The implicit cost of capital may, therefore, be defined as the rate of return associated with the best investment opportunity for the firm and its shareholders (or their consumption opportunities), that would be foregone, if the project presently under consideration by the firm were accepted. In other words, the implicit cost of capital is an opportunity cost and enters into consideration when alternative opportunities for the use of the firm's limited funds are available.

It should be noted that explicit costs arise when funds are raised. Implicit capital costs do not arise until funds are raised. Implicit capital costs do not arise until funds are invested or otherwise used. This is so because they represent alternative uses for the available funds. Therefore, implicit costs of capital exist regardless of the source of funds used.

The question is then how should the explicit and implicit costs of capital be integrated into mineral-investment valuation? The explicit cost of capital is the cut-off rate, or floor, below which no mineral investment is attractive. This does not necessarily mean that the firm should undertake all investments offering a rate of return greater than the explicit cost of capital. This is so because it is also essential to consider the implicit cost of capital involved in a particular use of funds, measured by the rate of return of the best opportunity foregone if the project under appraisal were accepted.

To illustrate this point let us suppose that the explicit cost of capital for financing a mining venture is found to be 10%. A DCF analysis of the proposed investment yields a rate of return of 12%. Furthermore, alternative investment opportunities in the economy offer 15%, ie, the implicit cost of capital is 15%. If management is assumed to behave rationally, the mine venture should not be undertaken. The discussion that follows examines models used in arriving at the marginal explicit costs of various sources of financing. More specifically, the discussion centres around determining the explicit costs of capital for debt, preferred stock, and equity.<sup>19</sup>

<sup>17</sup> This is similar to solving for  $\rho$  in the internal-rate-of-return approach.

<sup>18</sup> See William G. Lewellen, *The Cost of Capital*, Wadsworth Publishing Co, Inc, Belmont, California, 1969, p 87.

<sup>19</sup> In addition to these financing sources other sources are often used, such as convertible securities. For a more detailed discussion of convertible securities see James C. Van Horn, *The Function and Analysis of Capital Market Rates*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1970, pp 169-71.

### Cost of capital for specific sources of financing

**Cost of debt:** The explicit cost of debt can be determined by solving the following equation for  $r_i$ :

$$I_o = \frac{c_1}{(1+r_i)} + \frac{c_2}{(1+r_i)^2} + \dots + \frac{c_n}{(1+r_i)^n} + \frac{p_n}{(1+r_i)^n} \quad (13)$$

where

$I_o$  = net proceeds from the debt issue;

$c_t$  = cash outflows in period  $t$  in form of interest payments;

$p_n$  = principal to be returned to bondholders at the end of the duration of the loan;

$r_i$  = cost of debt before tax.

When the net proceeds for a bond equal its face, or par value (usually \$1000), the after-tax cost of debt is  $r_i^*$ , found as

$$r_i^* = r_i (1 - y) \quad (14)$$

where  $y$  is the marginal tax rate. This is so because interest payments are recognised as a business expense, and thus are tax deductible.

If the proceeds obtained for a bond differ from its face value, ie, the bond is sold at a premium or a discount, the use of Equation 14 is inappropriate. This is so because the premium or discount should be amortised for federal income tax purposes. In such a case the after-tax cost of debt is found as

$$r_i^* = \frac{2[c_t + \frac{1}{n}(P - I_o)](1 - y)}{(P + I_o)} \quad (15)$$

where

$r_i^*$  = after-tax cost of debt;

$P$  = face value of the bond (usually \$1000);

$I_o$  = proceeds per bond;

$y$  = marginal tax rate;

$n$  = duration of loan;

$\frac{1}{n}(P - I_o)$  = amortisation over the life of the bond;

$\frac{1}{2}(P + I_o)$  = average amount outstanding at any point in time over the duration of the loan.

Now let us suppose a case in which a firm has the policy of maintaining a given proportion of debt in its capital structure. In other words, debt is never really paid off. This is usually done by retiring outstanding bonds with the issuance of new debt with the same, or similar terms. This process may be looked at as a perpetual constant debt. Let us also assume that interest payments on the new issue are the same as on the old. If we now multiply both sides of Equation 13 by  $(1+r_i)$  and also exclude  $P_n/(1+r_i)^n$  since the principal is never really paid off, we get

$$I_o (1+r_i) = c + \frac{c}{(1+r_i)} + \dots + \frac{c}{(1+r_i)^{n-1}} \quad (16)$$

Subtracting Equation 13 from Equation 16 yields

$$I_0^*(1 + r_i) - I_0 = c - \frac{c}{(1 + r_i)^n} \quad (17)$$

or,

$$I_0 r_i = c - \frac{c}{(1 + r_i)^n}$$

As  $n$  approaches infinity  $c/(1 + r_i)^n$  approaches 0. Thus

$$I_0 r_i = c \quad (18)$$

and

$$r_i = \frac{c}{I_0} \quad (19)$$

The after-tax cost of debt then becomes

$$r_i^* = \frac{c}{I_0} (1 - y) \quad (20)$$

**Cost of preferred stock:** Preferred stocks have no maturity date. Most corporations that issue preferred stock plan on paying the stated dividends. Substituting in Equation 13  $D$  (stated annual dividends) for  $c$ ,  $r_p$  for  $r_i$  and, again, eliminating  $P_n/(1 + r_p)^n$  since preferred stocks are equity and as such the principal is never repaid,  $I_0$  is now the proceeds obtained from the sale of a preferred stock. To arrive at the after-tax cost of preferred stock we follow the rationale as applied to Equations 16 through 19. The cost of preferred stock  $r_p$  is

$$r_p = \frac{D}{I_0} \quad (21)$$

Note that since preferred stock dividends are not tax deductible, the cost of preferred stock is not affected by the tax rate, ie, the before- and after-tax costs of preferred stock are the same.

**Cost of equity (common stocks):** In theory, the cost of equity,  $r_e$ , is defined as the minimum rate of return that the firm must earn on the equity-financed portion of an investment project in order to leave unchanged the market price of the stock. In other words, the cost of new common stocks to a firm is related to the current market value of its common stock. In practice, the cost of common stocks is by far the most difficult cost to measure. This is so because the models employed in estimating the cost of equity capital are very sensitive to expectations on the part of the shareholders, ie, expectations regarding future dividend payments, future earnings per share, etc.

Two approaches to arrive at the cost of equity capital are presented below. The first is based on a dividend valuation model of common stock and looks at individual securities without considering

equilibrium conditions in the market for common stock. The second arrives at the cost of equity capital for a particular security by considering market equilibrium conditions, using the capital-asset pricing model.

*Cost of equity capital derived from a dividend valuation model.*<sup>20</sup>

From a theoretical standpoint, the market value of a share of stock to investors can be viewed as the present value of the expected stream of income (cash dividends) paid to them for an infinite number of years. This assumption may be justified on the ground that a common stock does not have a maturity date. At time 0 the value of a share of common stock is

$$P_0 = \frac{D_1}{(1+r_e)} + \frac{D_2}{(1+r_e)^2} + \frac{D_3}{(1+r_e)^3} + \dots + \frac{D_\infty}{(1+r_e)^\infty} \quad (22)$$

or

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_e)^t} \quad (23)$$

where

$P_0$  = value of a share of stock at time 0;

$D_t$  = dividend per share expected to be paid in period  $t$ ; and

$r_e$  = cost of equity capital.

Let us assume that dividends per share are expected to grow at a constant rate,  $g$ , in the form of

$$D_t = D_1 (1+g)^{t-1} \quad (24)$$

where  $D_1$  is the dividend per share expected to be paid at the end of period 1. Assuming further that  $r_e > g$ , we find that

$$P_0 = \frac{D_1}{(1+r_e)} + \frac{D_1(1+g)}{(1+r_e)^2} + \frac{D_1(1+g)^2}{(1+r_e)^3} + \frac{D_1(1+g)^3}{(1+r_e)^4} + \dots + \frac{D_1(1+g)^{\infty-1}}{(1+r_e)^\infty} \quad (25)$$

and

$$P_0 = \frac{D_1}{r_e - g} \quad (26)$$

The cost of equity capital,  $r_e$ , is therefore equal to

$$r_e = \frac{D_1}{P_0} + g \quad (27)$$

<sup>20</sup> This discussion is based on the Gordon dividend model. For a more detailed analysis of dividend models see Mayron J. Gordon, *The Investment, Financing and Valuation of the Corporation*, Irwin, Homewood, Ill., 1962; and Merton H. Miller and Franco Modigliani, 'Dividend policy, growth, and the valuation of shares,' *Journal of Business*, Vol 34, October 1961, pp 411-433.

Since, again, dividend payments are not tax deductible, the after-tax cost of equity is equal to the before-tax cost of equity.

*Cost of equity capital derived from the capital-asset pricing model:* The capital-asset pricing model has been developed on the foundations of portfolio theory as advanced by Markowitz.<sup>21</sup> The work of Markowitz provides a cornerstone for understanding some of the issues associated with the capital markets, as well as capital budgeting problems under conditions of risk. It deals with a set of efficient portfolios, given a set of expected returns, variances, and covariances of return for risky securities. Those efficient portfolios have the maximum expected portfolio return for a specified portfolio variance (when the variance of return is perceived as the risk associated with a portfolio), or the minimum variance for a specified expected return from the portfolio. Markowitz applies the expected utility criterion. According to this criterion, investors choose as their optimal portfolio the one whose combination of expected return and variance yields a maximum expected utility.

According to the capital-asset pricing model, the expected return of an individual security as related to the market as a whole is<sup>22</sup>

$$\bar{R}_j = f + \left( \frac{\bar{R}_m - f}{\sigma_m^2} \right) (\rho_{jm} \sigma_j \sigma_m) \quad (28)$$

or,

$$\bar{R}_j = f + (\bar{R}_m - f)\beta_j \quad (29)$$

where

$\bar{R}_j$  = expected value of return for security  $j$  where returns are calculated as  $\bar{R}_j = (p_{jt} + P_{j,t-1} + D_{jt})/P_{j,t-1}$ .  $P_{jt}$  is the market value of security  $j$  at time  $t$ , and  $D_{jt}$  is the dividend paid in time  $t$ ;

$f$  = risk-free rate of return. This rate is usually obtained from the yields on governmental bonds;

$\bar{R}_m$  = expected value of return for the market portfolio;

$\sigma_n^2$  = variance of return on the market portfolio;

$\rho_{jm}$  = correlation coefficient between the returns of security  $j$  and the returns on the market portfolio;

$\sigma_j$  = standard deviation of return for security  $j$ ;

$\sigma_m$  = standard deviation of returns for the market portfolio;

$\rho_{jm} \sigma_j \sigma_m$  = covariance of returns for security  $j$  with those of the market portfolio, portfolio  $m$ ;

$$(\rho_{jm} \sigma_j \sigma_m) / \sigma_m^2 = \beta_j$$

<sup>21</sup> A detailed discussion of the Markowitz foundations as well as the development of the capital-asset pricing model are beyond the scope of the present paper. For additional derivations and information see, H. Markowitz, 'Portfolio selection,' *The Journal of Finance*, Vol 7, March 1952, pp 77-9; Markowitz, *Portfolio Selection: Efficient Diversification of Investments*, John Wiley and Sons, Inc, New York, 1959; William F. Sharpe, 'Capital asset prices: A theory of market equilibrium under conditions of risk,' *Journal of Finance*, Vol 14, September 1946, pp 425-442; Sharpe, *Portfolio Theory and Capital Markets*, McGraw-Hill Book Co., New York, 1970, Chapters 2-6.

<sup>22</sup> For a discussion on this derivation, see Michael C. Jensen, 'Risk, the pricing of capital assets, and the evaluation of investment portfolios,' *Journal of Business*, Vol 42, April 1969, 176-77.

As can be seen, the larger the covariance of a security with the market portfolio, the greater the risk and the higher the expected return that is required, and vice versa. Thus, in a market equilibrium condition the capital-asset pricing model implies an expected return-risk relationship for all individual securities. A further examination of Equation 29 reveals that when a firm is using new equity financing for undertaking a new project this project must earn at least  $\bar{R}_j$  so as to leave the price of security  $j$  unchanged. In other words,  $\bar{R}_j$  is the

cost of equity for security  $j$ . This is so because if a project financed through the issuance of common stock  $j$  fails to earn  $\bar{R}_j$ , investors will demonstrate their dissatisfaction by unloading their holdings in stock  $j$ . This widespread sale of stock  $j$  is bound to decrease the market value of  $j$ . If we further believe that the objective of management is to maximise its owners' wealth, ie, maximising shareholders' wealth, which is equivalent to the maximisation of the market value of their stocks, then a decrease in the market value of  $j$  is unacceptable.

The cost of equity capital for security  $j$  in a market context can be estimated from Equation 29. The first step is to select a broad-based market index, such as Standard & Poor's 500-Stock Index. The second step is to calculate the actual *ex post* returns (monthly, quarterly, etc) for security  $j$ , less the risk-free rate. A similar procedure for the same horizon interval should be followed for the market *ex post* returns. The actual returns on security  $j$ , less the risk-free rate, in each period are then regressed against the returns for the market, less the risk-free rate. This yields a regression equation in the form of

$$R_{jt} - f_t = (R_{mt} - f_t) \beta_j \quad (30)$$

where

- $R_{jt}$  = return for security  $j$  in period  $t$ ;
- $f_t$  = risk-free rate in period  $t$ ;
- $\beta_j$  = regression coefficient;
- $R_{mt}$  = return for the market index in period  $t$ .

The regression analysis of Equation 30 estimates the regression coefficient,  $\beta_j$ . Next, substituting in Equation 29  $r_{ej}$ , the cost of capital for security  $j$  for  $\bar{R}_j$ , and using the mean risk-free rate for the period,  $\bar{f}$ , Equation 29 becomes:

$$r_{ej} = \bar{f} + [\bar{R}_m - \bar{f}] \beta_j \quad (31)$$

where

- $r_{ej}$  = cost of equity capital for security  $j$ ;
- $\bar{f}$  = arithmetic mean of  $f_t$  that prevailed over the sample period;
- $\bar{R}_m$  = mean of  $R_{mt}$  realised over this period.

Finally, by plugging in Equation 31 the value for  $\beta_j$  obtained from Equation 30 and the value for  $\bar{f}$ , we are able to solve for  $r_{ej}$ .

It should also be noted that Merrill, Lynch, Pierce, Fenner and Smith; Value Line Investment Survey, and others, offer information regarding regression coefficients,  $\beta_j$ , known as betas, for companies whose stock is actively traded on the major exchanges and over the counter. These calculations are usually based on monthly and quarterly data.

### Weighted average cost of capital

Since the concept of cost of capital is associated with the marginal cost of raising new capital to make an incremental (at the margin) investment in new projects, the weighted average cost of capital must

<sup>23</sup> See Raymond R. Reily and William E. Wecker, 'On the weighted average cost of capital,' *Journal of Financial and Quantitative Analysis*, January 1973, pp 123-126.

To demonstrate this point assume a firm with a capital structure which consists of common stock and debt only. From Equation 12 we know that the true cost of capital for a particular source of financing is the market-determined rate, which equates the net amount of funds received by the firm at time 0, with the discounted flow of net future payments to the source. Consequently, the cost of equity capital,  $r_e$ , should be determined as the discount rate that solves:

$$E_0 = \frac{D_1}{(1+r_e)} + \frac{D_2}{(1+r_e)^2} + \dots + \frac{D_n}{(1+r_e)^n} \quad (33)$$

where

- $E_0$  = per share equity capital received by the firm at time 0;
- $D_t$  = anticipated per share dividend payments to shareholders.

From an earlier discussion we know that the cost of debt can be determined from Equation 13 by solving it for  $r_f$ . Finally, the true overall cost of capital for the firm,  $r$ , can be found by solving for  $r$  in

$$E_0 + I_0 = \frac{(C_1 + D_1)}{(1+r)} + \frac{(C_2 + D_2)}{(1+r)^2} + \dots + \frac{(C_n + D_n + P_n)}{(1+r)^n} \quad (34)$$

where all terms are as defined earlier in the paper.

Solving Equations 33, 13 and 34 yields costs for  $r_e$ ,  $r_f$ , and  $r$ , respectively. Also, it was established in Equation 32 that the weighted average cost of capital for a firm,  $r^*$ , is the weighted sum of the costs of the individual capital sources. From a mathematical standpoint, if a polynomial has an order of  $n > 1$ , the root(s) of this polynomial cannot, in general, be expressed as a weighted average of any of the roots of an appropriate decomposition, ie,

$$r \neq r_e \left( \frac{S}{S+L} \right) + r_f \left( \frac{L}{S+L} \right) \quad (35)$$

For a formal proof of this statement see Reily and Wecker, pp 125-126. or,

$$r \neq r^* \quad (35a)$$

It is, therefore, argued that unless a single period payment is used, the use of a weighted average cost of capital may lead  
*continued p.295*

be a function of proportions of financing inputs the firm intends to employ. The weighted average cost of capital is exactly what its name implies – an average of the expected future costs of each of the sources of funds to be employed by the firm, properly weighted by the proportion they constitute in the financing package of the new project. The weights should be based on current market values of the financing package rather than the historical book values. The weighted average cost of capital is then obtained:

$$r^* = r_e \left( \frac{S}{S+L+P} \right) + r_i^* \left( \frac{L}{S+L+P} \right) + r_p \left( \frac{P}{S+L+P} \right) \quad (32)$$

where

- $r^*$  = weighted average cost of capital for the firm;
- $r_e$  = cost of equity capital;
- $r_i^*$  = after-tax cost of debt;
- $r_p$  = cost of preferred stock;
- $S$  = market value of common stock at the time of  $r^*$  being calculated;
- $L$  = market value of debt at the time of  $r^*$  being calculated;
- $P$  = market value of preferred stock at the time of  $r^*$  being calculated;
- $S+L+P$  = market value of the firm at the time of  $r^*$  being calculated.

The weighted average cost of capital is still a very controversial issue. It has been criticised on several grounds. One of these is that, due to a mathematical error, the weighted average cost of capital does not represent the true overall cost of capital.<sup>23</sup>

Findlay suggests that the contention that the weighted average cost of capital may only be employed in conjunction with perpetuities is incorrect. He shows, algebraically, that when, in Arditti's formulation, depreciation charges (which are a component of the firm's cash flow), are added to the earnings before interest and taxes, the weighted average cost of capital can be employed without requiring perpetuity of earnings or infinite firm's life.<sup>24</sup> Arditti also contends that in the computation of the weighted average cost of capital 'the appropriate after-tax costs of equity and debt are, respectively, the before-tax required rate of return on equity times one minus the corporate income tax rate and the interest rate.'<sup>25</sup> This contention contradicts the generally accepted theory that the after-tax weighted average cost of capital is determined according to Equation 32, ie,

$$r^* = r_e \left( \frac{S}{S+L} \right) + r_i (1-y) \left( \frac{L}{S+L} \right) \quad (36)$$

where both  $r_e$  and  $r_i$  are before-tax cost of equity and debt, respectively. Arditti argues that  $r^*$  should be calculated in the following manner:

$$r^* = r_e (1-y) \left( \frac{S}{S+L} \right) + r_i \left( \frac{L}{S+L} \right) \quad (37)$$

Arditti's economic reasoning for Equation 37 is that if we assume away personal taxes, the after-tax cost of debt must be equal to  $r_i$ . This is so, he argues, because the shareholders can make  $r_i$  by simply investing their funds in a savings account. On the other hand, the earnings generated by the corporation are subject to income tax, the after tax minimum expected rate of return, ie,  $r_e^*$ , is equal to  $r_i(1 - y)$ . This contradiction with the accepted cost of capital theory is attributed to different definitions concerning the tax shelter on debt. It can be shown that when the definitions for the tax shelter in Arditti's formulation and the accepted theory are reconciled, then both models yield the same equation for the weighted average cost of capital, ie, Equations 32 or 36.<sup>26</sup>

### Weighted average cost of capital: Example

The Coal Mining Company has the following capital structure as of December 31, 1975 (in dollars):

Debt (6 1/2%)		30 000 000
Preferred stock (7 1/2%)		10 000 000
Common stock	10 000 000	
Retained earnings	30 000 000	
Common equity		40 000 000
Total capitalisation		80 000 000

to the establishment of an incorrect investment cut-off rate. However, it can be shown that  $r = r^*$  if the firm is expected to earn a perpetual constant amount per annum. For formal proofs and a more detailed discussion, see for example, Fred D. Arditti, 'The weighted average cost of capital: Some questions on its definition, interpretation, and use,' *Journal of Finance*, September 1973, pp 1001-7; and S.C. Myers, 'Interactions of corporate financing and investment decisions - Implications for capital budgeting,' *Journal of Finance*, March 1974, pp 1-25. Arditti and Myers differ, however, with respect to the value for earnings. Arditti refers to before-tax, and before interest earnings, whereas Myers refers to after-tax, and after interest earnings. Arditti also claims that if, on the other hand, earnings are not expected to be constant forever, or the firm has a finite life, or both, then the firm's expected earnings stream is no longer a perpetuity and then inequality 35 holds (pp 1002-1003).

<sup>24</sup> M. Chapman Findlay III, 'The weighted average cost of capital,' *Journal of Finance*, June 1975, pp 879-880.

<sup>25</sup> Arditti, *op cit*, Reference 23, p 1005.

<sup>26</sup> See for example, R. Richardson Pettit, 'The weighted average cost of capital: Some questions on its definitions, interpretation and use: Comment,' *Journal of Finance*, June 1975, pp 881-882; John J. McConnell and Carl M. Sandberg, 'The weighted average cost of capital: Some questions on its definitions, interpretation and use: Comment,' *Journal of Finance*, June 1975, pp 887-889.

Earnings per share have grown steadily from \$1.54 in 1968 to \$3.00 estimated for 1975. The investment community, expecting this growth to continue, applies a price/earning ratio of 20 to yield a current market price of \$60.00 per share. Coal Mining Company is expected to pay in 1976 an annual dividend per share of \$2.00. Future dividend payments are expected to grow at the same rate as earnings.

The Coal Mining Company is considering the development of a mine in the state of Montana. The company plans to acquire the funds needed to undertake this project through the issuance of bonds, preferred stocks, and common stocks. The proposed proportions of these financial instruments in the financing package for the project is planned to be the same as their proportion in the capital structure of the company as of December 31, 1975.

The management of Coal Mining Company expects to float the following financial instruments and their respective yields:

**Bonds:** par value equals market value of \$1000 offer a yield of 8 1/2%.

**Preferred stock:** par value equals market value of \$100 offering a yield of 9 1/2%.

**Common stock:** market value of \$60/share.

Assuming also that the company is in the 50% tax bracket, what is the weighted average cost of capital associated with the financing of the proposed project?

*Calculation of market weights:*

	\$	% of total
Debt	30 000 000	37.5
Preferred	10 000 000	12.5
Equity	40 000 000	50.0
	80 000 000	100

*Cost of capital for specific sources:*

(i) after tax – cost of debt:

$$r_i^* = r_i (1 - y) = 0.085 (1 - 0.50) = 0.0425$$

(ii) cost of preferred stock:

$$r_p = \frac{D}{I_o} = \frac{9.50}{100} = 0.0950$$

(iii) cost of equity:

If we assume a constant annual growth rate in dividends, the growth rate can be found from the following equation:

$$1.54 (1 + g)^7 = 3.00$$

and  $g = 0.10$ , or 10% annually.

$$r_e = \frac{D_1}{P_o} + g = \frac{2.00}{60.00} + 0.10 = 0.1330$$

*Calculations of the weighted average cost of capital:*

Instrument	Proportion	After-tax cost of capital	Weighted average cost of capital
Debt	0.375	0.0425	0.01593
Preferred stock	0.125	0.0950	0.01187
Common stock	0.500	0.1330	0.06650
Average cost of new capital			0.09430 or 9.43%

### Summary and conclusions

The Hoskold and Morkill models fail to account for the minimum required rate of return from an investment, ie, the cost of capital. The risky rate of return,  $r$ , usually employed in these models, is very often assigned an arbitrary value which does not consider the risk-return trade-off associated with undertaking a new investment. It does not account for the financing mix associated with raising the funds required for the new investments.

For both the net present value approach and the internal-rate-of-return method the cost of capital was a crucial factor in determining acceptance or rejection of an investment proposal. However, the mineral industry, by and large, does not seem to employ properly this important theoretical concept.

To arrive at the overall cost of capital for a firm at a given time, the costs of the specific sources of financing must be calculated first. Then, the weighted average cost of capital is determined on the basis of the expected future costs of each of the sources of funds to be employed by the firm, properly weighted by the proportion which they constitute in the financing package of the new project. The controversy over the theoretical properties of the weighted average cost of capital is far from being resolved. Nevertheless, the weighted average cost of capital still plays a crucial role in capital budgeting techniques. The prevailing view is that the weighted average cost of capital is still the best performance measure to be used as a hurdle, or cut-off rate in capital budgeting.<sup>27</sup>

<sup>27</sup> Even Arditti emphasises that his study does not analyse the appropriateness of the weighted average cost of capital as a cut-off rate in investment decisions. He focuses on its use in determining an optimal capital structure, ie, an optimal debt/equity ratio.